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BCA-404.

Equivalance Relation: A Relation R on a Set A is said to be Equivalent if R is
(i) Reflexive (ii) Symmetric (iii) Transitive

Example: Let $A = \{1,2,3\}$.

$R_1 = \{\phi\}$ ✗ $(1,1) \notin R_1 \Rightarrow R_1$ is not Reflexive.

$R_2 = \{(11),(22),(33)\}$ ✓

$R_3 = \{(11),(22),(12)\}$ ✗ $(3,3) \notin R_3$

$R_4 = \{(11),(22),(33),(12),(21)\}$. $(21) \notin R_4$ ✗

$R_5 = \{(11),(22),(33),(12),(21),(21)\}$. ✓

$R_6 = \{A \times A\}$ ✓

Ref: $\forall a \in A \Rightarrow (a,a) \in R$

Symm: if $(a,b) \in R \Rightarrow (b,a) \in$

Transitive: $\forall (a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$.

$R_7 = \{(a,b) : (a-b)$ is an even number where $a,b \in \mathbb{R}$ (real numbers)$\}$ ✓

$R_8 = \{(a,b) : (a-b)$ is an odd number$\}$ where, $a,b \in \mathbb{R}$. ✗

(Hint: '0' is an even number).

Case

$(a-a) = 0$

For R_8: $(a,a) \in R_8$ if $a-a = 0 =$ odd number ≠

$\Rightarrow (a,a) \notin R_8$

$\Rightarrow R_8$ is not Reflexive $\Rightarrow R_8$ is not P.O. Relation also Not an Equivalance Rel.

For R_7: $(a,a) \in R_7$ as $(a-a) = 0 =$ even number $\Rightarrow R_7$ is Ref.

Let $(a,b) \in R_7 \Rightarrow a-b =$ even $\Rightarrow (b-a) =$ even
$\Rightarrow (b,a) \in R_7$
$\Rightarrow R_7$ is Symmetric

Similarly R_7 is transitive as for example $a=2, b=4, c=6$
$\Rightarrow R_7$ is Equivalance Relation

Partial order Relation! A Relation R on a set A is said to be P.O.R. if it is
(i) Reflexive (ii) Antisymmetric (iii) Transitive

Example! Let $A = \{1,2,3\}$

$R_1 = \{\phi\}$ ✗

$R_2 = \{(11),(22),(12)\}$ ✗ $(3,3) \notin R_3$

$R_3 = \{(11)\ (22)\ (33)\}$ ✓

$R_4 = \{(11)(22)\ (33)\ (12)\}$ ✓

$R_5 = \{(11)(22)(33)\ (12)\ (21)\}$ ✗

$R_6 = \{(11)\ (22)\ (33)(13)(23)\}$ ✓

$R_7 = \{A \times A\}$ ✗ $(12) \in R_7$ $(21) \in R_7$

$R_8 = \{(a,b) : a,b \in \mathbb{Z}, a < b\}$
$(a,a) \in R_8$ if $a < a$ # $\Rightarrow R_8$ is not Ref.

$R_9 = \{(a,b) : a,b \in \mathbb{Z}, a \leq b\}$ ✓
$a \leq b \Rightarrow b \nleq a \Rightarrow R_9$ is Antisym
$a \leq b, b \leq c \Rightarrow a \leq c \Rightarrow R_9$ is transitive

$R_{10} = \{(a,b) : a,b \in \mathbb{Z}, \frac{b}{a} \in \mathbb{Z}\}$.

Antisymmetric
if $(ab) \in R$
$\Rightarrow (ba) \notin R$.
for $a \neq b$.

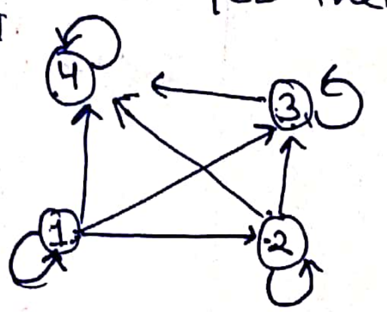
Hasse diagram: Pictorial Representation of Partial order Relation (POSET) is called Hasse diagram.

Example: Let $A = \{1, 2, 3, 4\}$.

$R = \{(1 1), (1 2), (1 3), (1 4), (2 2), (2 3), (2 4), (3 3), (3 4), (4 4)\}$.

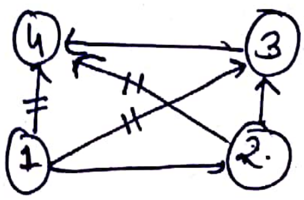
Step-I: Check the Given Relation is POSET or not. If Yes then proceed to next step.

Step-II

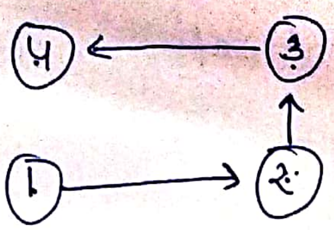


$a_i \to a_j$ if $(a_i, a_j) \in R$.

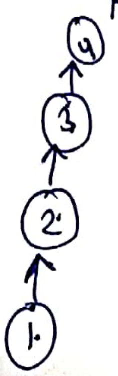
Step-III Remove all Reflexive edge. (all self loop)



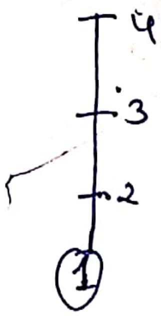
Step-IV Remove all transitive edge.



Step-V Represent the diagram in upward direction



or



Hasse diagram of R.

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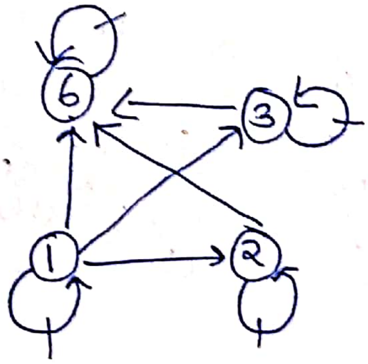
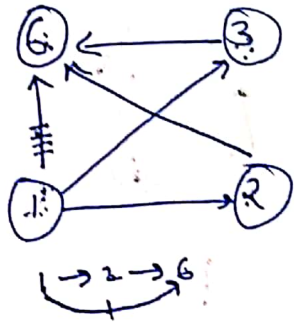
Example! $A = \{1, 2, 3, 6\}$. Draw Hasse diagram

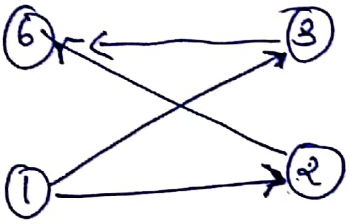
$R = \{(1 1), (1, 2), (1 3), (1 6), (2 2), (2 6), (3 3), (3, 6) (6 6)\}$



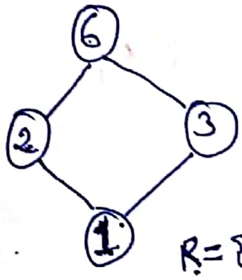
self loop → Remove Ref. edge



→ Remove Transitive edge.



→ upward direction



Hasse diagram.

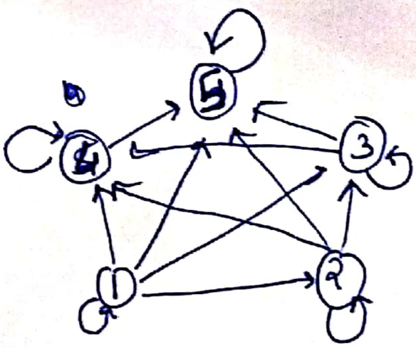
R = 22

$R = \{(1 1) (3 3) (6 6) (2, 2)$
$(1 3) (1 2) (1 6) (2, 6)$
$(3, 6)\}$

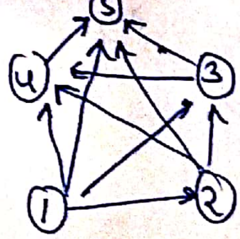
Example: POSET $(\{1, 2, 3, 4, 5\}, \leq)$, find Hasse diag. if exist.

(Note : Check the given Relation is (POSET) or not if yes then Hasse diagram exist otherwise not)

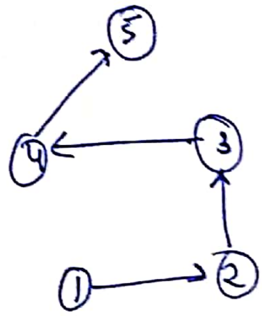
$R = \{(1 1), (1 2) (1 3) (1 4) (1 5)$
$(2 2) (2 3) (2 4) (2 5)$
$(3 3) (3 4) (3 5)$
$(4 4) (4 5)$
$(5 5)\}$



→



→



Now upword direction rep :

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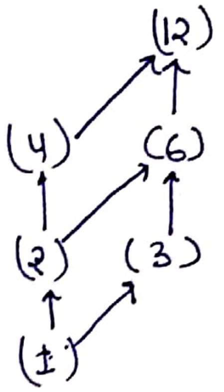
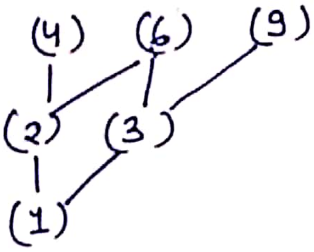
Example: $(\{1, 2, 3, 4, 6, 12\}, /)$

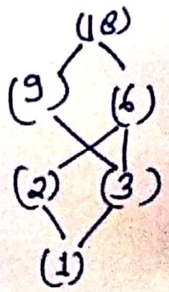


$R = \{(1\,1)\ (2\,2)\ (3\,3)\ (4\,4)\ (6\,6)\ (12;12),\ (1\,3)\ (1\,2)\ (2\,6)\ (2\,4)\ (3,6),\ (4\,12)\ (6\,12)\}.$

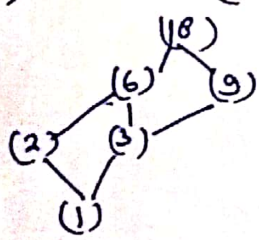
Example: $(\{1, 2, 3, 4, 6, 9\}, /)$



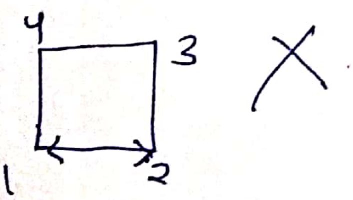
Example: $(\{1, 2, 3, 6, 9, 18\}, /)$ or $(D_{18}, /)$



or



Example: $(\{1, 2, 3, 4, 5\}, \leq)$



$R = \{(1\,1)\ (2\,2)\ (3\,3)\ (4\,4),\ (1\,2)\ (2\,1)\}$

Imp Note: Hasse diagram does not contain any Horizontal line.